A Methodology for Assessment of Geothermal Energy Reserves Associated with Volcanic Systems

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ABSTRACT

The potentially exploitable geothermal energy reserves associated with an active or dormant volcano can be estimated using a methodology that combines principles of conductive heat transfer and volcanology to calculate temperature distribution in time and space following magma emplacement, then calculates potentially recoverable geothermal energy reserves using principles of thermodynamics. Four principal magma characteristics must be estimated for this calculation: volume, depth, age and temperature of emplacement. Since these four parameters and the "recovery factor" (the fraction of *in-situ* thermal energy that is recoverable) are the most important uncertain parameters in such a reserves calculation, a probabilistic simulation is done by assigning to each parameter a reasonable maximum and a reasonable minimum value and a probability distribution, which is usually triangular (if a most-likely value can be defined) or uniform (all values between the minimum and maximum being equally likely). The mean, standard deviation and most-likely values of reserves are then calculated statistically through Monte Carlo sampling of the uncertain parameters. The bases for assigning these maximum and minimum values are discussed, and an example of this methodology applied to a volcano is presented. In a nationwide assessment of geothermal prospects in Nicaragua, this methodology has been applied to 14 different volcanoes, with magma bodies that range in volume from 4 to 220 cubic km, depth from 3 to 7 km, age from 5,000 to 500,000 years, and temperature from 900°C to 1,100°C. With a uniform distribution of 0.05 to 0.1 for the heat recovery factor, a typical "utilization factor" (the fraction of thermodynamically available work that can be converted to electricity) of 45% and a power plant capacity factor of 90%, the mean calculated reserves per volcano ranged from 61 MW to 676 MW for 30 years. In the absence of detailed exploration and drilling, this methodology provides a perfectly general and internally consistent approach to estimating at least the upper limit of geothermal reserves in the volcanic environment.

Introduction

A geothermal reservoir is often associated, genetically and spatially, with a volcano or volcanic complex that constitutes the source of geothermal heat. Once the region around such a volcano has been adequately explored, with deep wells included, to define the subsurface temperature distribution and the volume of the reservoir(s), geothermal energy reserves associated with the volcano can be estimated (for example, see Brook, *et al*, 1978). However, most volcanoes in the world lie unexplored except for perhaps surface geological investigations. No standard methodology exists for quantifying the potential reserves of exploitable geothermal energy associated with unexplored or inadequately explored volcanoes. Developers are reluctant to invest in extensive exploration of a volcanic complex unless the potential geothermal energy reserves associated with the complex are indicated to be large enough to be attractive; however, the reserves associated with a volcanic complex cannot be estimated without adequate exploration and drilling. We have repeatedly faced this conundrum while conducting nationwide assessments of geothermal resources in countries like Japan and in Central America, which are dotted with active and dormant volcanoes. As a solution to this problem we developed the proposed methodology, which is more rigorous and general than the pioneering approach of Smith and Shaw (1975; 1978). We developed this methodology in 1987, in connection with a nationwide heat source assessment program in Japan.

Magmatic Heat Transfer Calculation

In a volcanic geothermal system the ultimate heat source is the magma emplaced at relatively shallow levels beneath the ground surface as part of the process of volcanic activity. Following its emplacement, the magma body gradually heats up the surrounding rock by conduction as it cools and crystallizes to form a body of intrusive rock. The reserves of heat energy around the magma body gradually become concentrated through convective heat transfer by fluids circulating through faults and fractures in the surrounding rock. Such faults and fractures are often the result of stresses induced by magma emplacement. This convective heat transfer eventually gives rise to geothermal reservoirs, localized in porous and permeable pockets of subsurface rock, in the vicinity of the volcano. Obviously, the sum total of energy reserves in all the geothermal reservoirs within the heated zone around the volcano cannot exceed the total energy reserves within this heated zone. The recoverable energy reserves associated with a single volcano or volcanic complex can thus be represented as a fraction of the total energy reserves in the heated zone around the volcano, which can be approximated from the knowledge of the characteristics of the magma body, as discussed below. Therefore, even though no individual reservoir may yet be identified or defined within a volcanic area, at least the upper limit of geothermal energy reserves can still be estimated.

Conductive heat transfer from a magma body to the surrounding rock can be calculated if one can estimate the following basic parameters of the magma: volume, depth of burial, age and initial temperature. Although the shape of the magma body also impacts this heat transfer, it is usually much less important than the other four variables in the heat transfer calculation for relatively equidimensional magma shapes (such as approximate cubes or spheres). The shape becomes critical in such calculations only if one or two dimensions are far more prominent than the others, such as in dikes, sills or narrow plugs. The properties of the rock surrounding the magma are relatively well known compared to the magma characteristics; therefore, assumed values of rock properties typical of volcanic systems can suffice for this calculation. The calculation also requires the assumption of a set of initial and boundary conditions for the magma.

Three idealized types of magma bodies are commonly recognized: (a) "instantaneous source" or cooling magma (following its emplacement, the magma body cools down continuously); (b) constant-temperature magma (there is so much

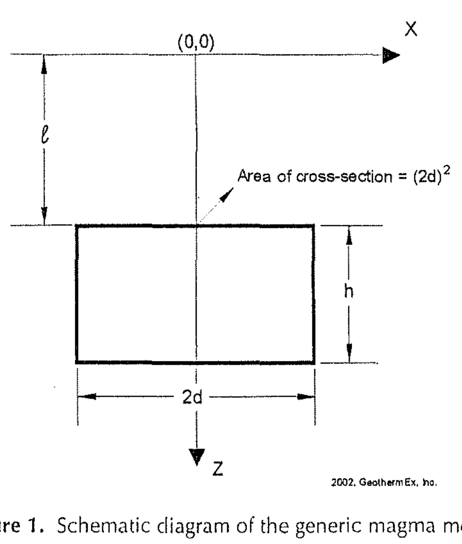

Figure 1. Schematic diagram of the generic magma model.

convection within the magma body that it retains its original temperature indefinitely); and (c) "continuous source" or constant heat discharge magma (due to continuous convection within the magma, the rate of heat discharge from the magma body remains constant with time). One of these three idealizations needs to be invoked in order to solve the differential equation describing the conductive heat transfer process from magma; the first idealization being more conservative than the other two. One of the previously developed mathematical solutions (for example, Carslaw and Jaeger, 1959; Lovering, 1935) can be utilized for calculating the temperature distribution around the magma body depending on the idealization.

For this paper we have used the solution for a cooling magma as described below (Figure 1). If a parallelepiped magma body of square horizontal cross-section of width $4d^2$ and height h, at an initial temperature T_o has its top at a depth l below the ground surface (maintained at zero temperature for t ≥0), and if the subsurface had an initial temperature of zero everywhere, then the temperature T at time t at any point a horizontal distance x away from the center of the body and at a depth z is given by:

$$T(x,z,t) = \frac{T_o}{4} erf\left(\frac{d}{2\sqrt{\alpha t}}\right)\left[erf\left(\frac{x+d}{2\sqrt{\alpha t}}\right) - erf\left(\frac{x-d}{2\sqrt{\alpha t}}\right)\right] \cdot \left\{erf\left(\frac{h+\ell-z}{2\sqrt{\alpha t}}\right) - erf\left(\frac{h+\ell+z}{2\sqrt{\alpha t}}\right) - erf\left(\frac{\ell-z}{2\sqrt{\alpha t}}\right) + erf\left(\frac{\ell+z}{2\sqrt{\alpha t}}\right)\right\} \quad (1)$$

where

α = thermal diffusivity = K/ρc, (2)

K = thermal conductivity of surrounding rock,

c = specific heat of surrounding rock,

ρ = density of surrounding rock, and

$$\text{erf}(x) = \text{Error Function} = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du\,. \quad (3)$$

This mathematical solution, which for simplicity ignores the latent heat of magma solidification, can be used to calculate the distribution of temperature as a function of time and distance from the magma body. Ignoring the latent heat does not greatly alter the temperature distribution around the magma body (Jaeger, 1961). The initial temperature at all points before magma emplacement must, however, be added to these temperatures. The initial temperature distribution is given by the regional vertical temperature gradient that existed before magma intrusion. Thus, one can approximate the temperature at any depth under a surface location, at any distance from the magma chamber, at any time after magma emplacement.

Figures 2 through 5 show examples of such calculations for a magma body with a square horizontal cross-section of 121 km² and a thickness of 6 km, the pre-existing regional vertical temperature gradient being 60°C/km. Figures 2 and 3 show the calculated temperature at various depth levels as a function of time for distances of 0 km and 8 km, respectively, from a magma body

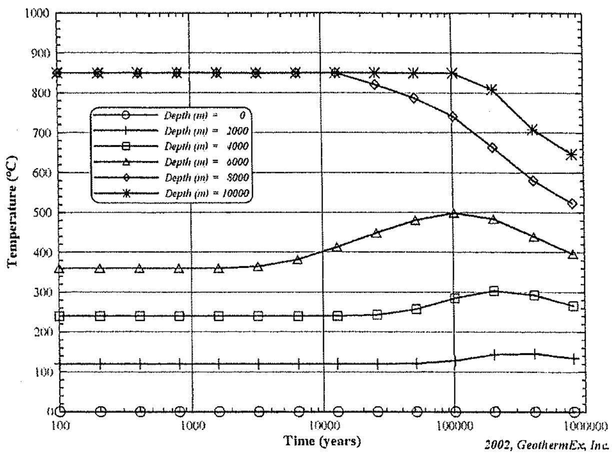

Figure 2. Calculated temperature versus time at 0 km distance.

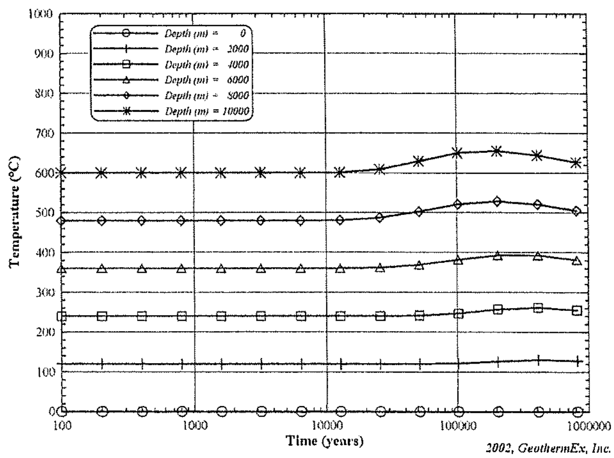

Figure 3. Calculated temperature versus time at 8 km distance.

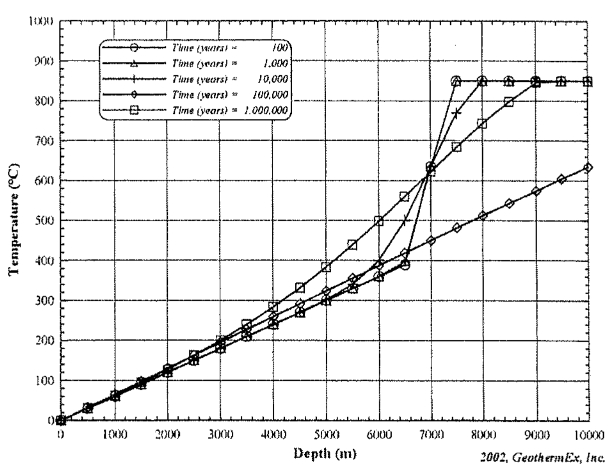

Figure 4. Calculated temperature versus depth at 0 km distance.

initially at 850°C and emplaced at a depth of 7 km. The thermal diffusivity has been assumed to be $0.93 \times 10^{-6} m^2/s$. Figure 2 shows that within the typical drillable depth range (up to 4 km), subsurface temperatures do not show any elevation until about 25,000 years have passed since magma emplacement, while at a depth

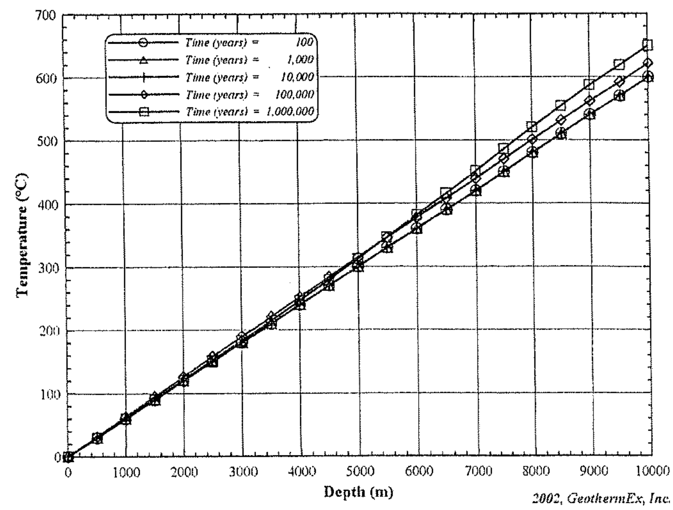

Figure 5. Calculated temperature versus depth at 8 km distance.

of 6 km, temperature elevation begins only about 1,500 years after magma emplacement. Similarly, Figures 4 and 5 show the calculated temperature versus depth at various times at distances of 0 km and 8 km, respectively, from the same magma body.

Reserve Estimation

From the calculated temperature distribution around the magma body, one can estimate the energy reserves underneath any given prospect within a given depth range. For the purposes of the reserves estimations herein we assume a depth limit of 4 km. We also assume a cut-off average temperature of 200°C; that is, any subsurface rock volume at a temperature of less than 200°C is considered non-commercial and outside the commercial "reservoir". Finally, we typically assume a vertical subsurface temperature gradient of 50°C/km before magma emplacement. These assumptions are conservative and can be relaxed where warranted.

One can then calculate the geothermal reserves per unit area (of the ground surface of the prospect) associated with a magma body from the calculated temperature distribution. Our approach to this calculation, which is an extension of the concepts originally presented in Brook, *et al* (1978), is as follows:

$$E = dc_v(T-T_o) \cdot R/F/L, \tag{4}$$

Where

- E = MW reserves per km^2 at a distance x from the center of the caldera,
- d = the depth down to which the energy reserves are to be estimated,
- c_v = volumetric specific heat of the reservoir,
- T = calculated average temperature (in absolute unit) between the ground surface and depth d at a distance x from the center of the caldera,
- T_o = rejection temperature in absolute unit (equivalent to the average annual ambient temperature),
- F = power plant capacity factor (the fraction of time the plant produces power on an annual basis),

R = overall recovery efficiency (the fraction of thermal energy-in-place in the reservoir that is converted to electrical energy at the plant), and

L = power plant life.

The parameter c_v in (4) is given by:

$$c_v = \rho_r c_r (1-\phi) + \rho_f c_f \phi \tag{5}$$

where

ρ_r = density of rock matrix,
c_r = specific heat of rock matrix,
ρ_f = density of reservoir fluid at temperature T,
c_f = specific heat of reservoir fluid at temperature T, and
ϕ = reservoir porosity.

The parameter R in (4) can be represented as:

$$R = \frac{W.r.e}{\overline{c_f}(T - T_o)} \tag{6}$$

where

r = recovery factor (the fraction of thermal energy-in-place that is recoverable at the surface as thermal energy),
$\overline{c_f}$ = average specific heat of reservoir fluid within the temperature range of T_o to T,
W = maximum thermodynamically available work from the produced fluid, and
e = utilization factor, which accounts for mechanical and other energy losses that occur in a real power cycle.

The parameter W in (6) is derived from the First and Second Laws of Thermodynamics:

$$dq = c_f dT, \text{ and} \tag{7}$$

$$dW = dq(1 - T_o / T) \tag{8}$$

In the actual calculation of reserves, depth d is subdivided into small intervals, and those interval units found to have one average temperature of less than 200°C are cast out.

Once one decides on a solution (for the assumed initial and boundary conditions) and estimates or assumes the required rock properties, the challenge is to estimate the four magma characteristics (volume, depth, age and temperature). All four are uncertain; therefore, we use a Monte Carlo simulation approach. For each trial we calculate the geothermal reserves per km^2 at a distance x from the center of the volcano by sampling from a probability density function of each of the five most uncertain variables, namely, magma volume, magma depth, magma age, magma temperature and recovery factor (r). We assume that only 10% to 20% of the heat energy (within rock shallower than 4 km and at temperatures higher than 200°C) around the magma body would be concentrated in geothermal reservoirs suitable for exploitation, and only 50% of this thermal energy would be recoverable at the wellhead. This results in a heat recovery factor (r) of 0.05 to 0.1, which is considerably lower than the value of 0.25 proposed by the U.S. Geological Survey in 1978 (Brook, *et al*, 1978); case histories of geothermal projects since 1978 have proven a value of 0.25 to be too optimistic. Based on our experience in both volumetric reserve estimation and numerical simulation of actual reservoir performance in numerous projects, we believe 0.05 to 0.10 to be a more reasonable range of values for the heat recovery factor. Although wells may be drilled to a depth shallower than 4 km, convection through faults and fractures will undoubtedly carry some of the heat from these depths to the producing wells. Therefore, reserve estimation by this method yields an upper limit of the potentially available reserves.

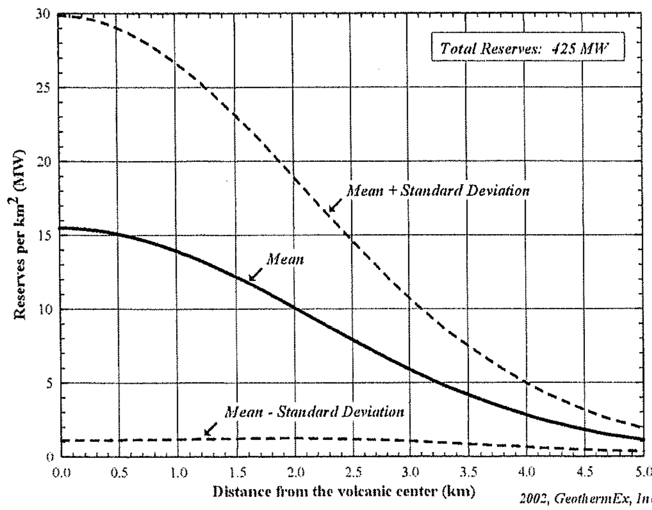

Figure 6. Calculated reserves per km^2 versus distance.

It should be noted that cooling magma does not always cause much elevation of temperature by conduction alone at drillable depths. However, the intrusion of a magma body (or multiple intrusions by magma bodies), induces stress in the country rock; this is manifested by fractures, which enable structurally-controlled hydrothermal convection. Attractively elevated temperatures at drillable depths are the result of upward convection of heat from the magma through circulating groundwater.

Figure 6 shows the calculated mean reserves per km^2, plus and minus one standard deviation, as a function of distance from the center of the volcano for the case of a cubic magma chamber with the following characteristics: 900° to 1,100°C initial temperature, 50 to 100 km^3 volume, 3 to 7 km emplacement depth, and an emplacement time of 30,000 to 60,000 years ago. It is clear from this figure that the magma's contribution to subsurface temperatures in this case becomes negligible at a distance greater than about 4 km from the center of the volcano. Similar plots can be prepared for any volcano by ascribing proper ranges of the magma parameters (depth, volume, age and temperature). It should be noted that the standard deviation of the calculated reserves would be far less if most-likely values were also specified; that is, if triangular probability distributions (rather than uniform distributions) were assigned for the uncertain variables. From the Monte Carlo simulation results, such as shown in Figure 6, one can calculate the total reserves related to the volcano as follows.

If E is reserves in MW/km^2 (from the ground surface down to a depth d) at a distance x km from the center of the volcano, then the total reserves in-place due to the magma body is given by:

$$\text{Reserves} = 2p \int_0^{x'} (xE)\,dx, \tag{9}$$

where x′ is the largest distance from the volcano's center at which magma has caused elevated temperatures.

If there are multiple volcanoes of different ages, depths, volumes, etc. within a given area, the total reserves can be approximated by invoking the Principle of Superposition, that is, by summarizing the reserves, given by equation (1), for all the magma bodies represented by the volcanoes.

Estimation of Magma Body Parameters

It is difficult to determine precisely the size, shape, position, depth and initial temperature of the magmatic complex beneath an active or young volcano. Certain techniques, including geophysical and petrological methods, can be applied to estimate these parameters. However, the precision that can be obtained by these methods is variable, and they require substantial detailed study and analysis, and major expenditures of exploration funds, in order to achieve a reasonable level of confidence. For this reason, to estimate the magma body parameters needed for calculating energy reserves, we often adopt methods that rely on readily observable characteristics of the volcanoes. Such methods may yield estimates that are less precise than could be achieved by more sophisticated techniques, but they have the advantage of providing a consistent objective and inexpensive basis for comparing the reserves associated with various volcanoes. For the volcanic complexes along the Volcanic Cordillera of Central America, our approach to the estimation of the magma body parameters is discussed below.

Size and Shape

Magma that ascends from its point of generation (typically in the mantle) may be emplaced beneath the surface, or erupted above the surface. Typically, the amount of magma that is erupted as lava or pyroclastic material is balanced by a roughly equal amount that is emplaced at shallow levels (a few km) as a magma body or intrusive rock. Therefore, the volume of the magmatic/intrusive complex beneath the volcano can be estimated approximately by determining the volume of associated extrusive material.

For many volcanoes, most of the erupted material may remain as part of the present-day volcanic edifice. This is particularly true of cone-shaped strato-volcanoes, where eruptive activity is dominated by lava flows and moderately explosive pyroclastic eruptions. In these instances, the volume of the volcanic edifice represents a good estimate of the minimum volume of the magmatic/intrusive complex that is available to act as a source of geothermal heat. The volume of such a volcano or volcanic complex can be estimated from detailed topographic maps to estimate the minimum magma volume.

Removal or dispersal of material by erosion, more highly explosive volcanism, or subsidence will tend to reduce the size of the volcanic edifice and cause under-estimation of the magma volume. Taking this into account, the maximum size of the magmatic/intrusive complex could be as much as twice the minimum size estimated from the volume of the volcanic edifice. Therefore, where magma volume has been estimated by this method, its probability distribution is assumed to range between one and two times the calculated volume of erupted material.

Estimation of magma volume by this method may be too conservative for more explosive, caldera-forming volcanoes because the nature of their activity tends to disperse material over a broader area, making accurate calculation of the volume of erupted products difficult. In addition, these volcanoes are more likely to have long-lived, well-developed magma chambers. For this reason, we use a different approach to estimating magma volume, based on caldera dimensions, where under-estimation by the eruptive-product method is likely.

A major caldera-forming eruption does not completely empty the magma chamber from which it originates. Various studies have shown that, instead, only about 10% to 20% of the magma from the uppermost part of the chamber is erupted. The size of the caldera formed by the collapse of the upper part of the chamber as a consequence of the eruption is approximately equal to the volume of the magma erupted. Therefore, the size of the magmatic complex can be estimated to be from about 5 to 10 times the volume of the most recently-formed caldera. We use these limits as the limits of the probability distribution of magma volume in the cases where the eruptive-products method is inapplicable.

As discussed before, the calculation of energy reserves is relatively insensitive to the shape of the magma body. Therefore, no attempt has been made to estimate the specific dimensions of the magmatic complex for a volcano; such an estimate would be highly speculative in any case. For a few volcanoes, where there is reason to infer that the distribution of magma may deviate significantly from a regular shape, a solution more appropriate than (1) can be used.

Depth

Magma that does not erupt to the surface will tend to accumulate at or near the depth where its density is in balance with the density of the surrounding rock; silicic magmas being of lower density tend to occur shallower than intermediate or mafic magmas. This depth is referred to as the level of neutral buoyancy, and is typically about 5 km below the surface. Some magma may be emplaced at shallower depths (as dikes or other small intrusive bodies), and the base of the magma chamber may be deeper, but the level of neutral buoyancy represents a useful estimate of the depth of the magma body. For the volcanoes we have investigated in Central America, the probability distribution of magma depth has been assumed to have limits of 3 km and 7 km, unless more specific information has been available.

Temperature

For volcanoes in Japan we have used magma temperature estimates from pyroxene geothermometry. The volcanoes of the Volcanic Cordillera of Central America share fairly similar petrologic characteristics that suggest similar conditions of magma genesis. The predominance of basaltic to andesitic volcanism in the Nicaraguan volcanoes (Nyström *et al.*, 1993) indicates that most, if not all, magma emplaced at shallow levels may be fundamentally basaltic. For volcanoes of this region, a range of 900° to 1,100°C has been assumed for the probability distribution of initial magma temperature, reflecting the typical temperature range for magma of this inferred composition.

Age

The age of magma emplacement can usually be estimated with more confidence than the size and shape of the complex. Sources of information for estimating age may include: (a) radiometric dating of eruptive products, (b) inferences from rates of eruptive activity (for example, McKnight, *et. al.*, 1997), and (c) comparison with other volcanoes whose ages are better known. The amount of information typically is sufficient to construct appropriate probability distributions for use in energy reserve calculations.

Example of Application of the Methodology

We have applied the methodology presented above to numerous volcanic complexes assuming the magma bodies to be cubic in shape. The results will be essentially the same if the magma body is assumed to be spherical or any other nearly-equidimensional shape. For example, for a volcano in Nicaragua, the following fixed parameters were assumed:

Maximum depth considered for reserve estimation	= 4 km
Cut-off resource temperature	= 200°C
Initial vertical temperature gradient	= 50°C/km
Thermal conductivity of rock	= 0.0025 kJ/m/s/°C
Specific heat of rock	= 1.0 kJ/kg/°C
Density of rock	= 2,700 kg/m^3
Porosity of rock	= 3%
Utilization factor	= 45%
Rejection temperature	= 30°C
Power plant capacity factor	= 0.90
Power plant life	= 30 years

The following uncertain parameters were estimated to have uniform probability distributions as follows:

Parameters	Minimum Value	Maximum Value
Magma volume (km^3)	50	100
Magma depth (km)	3	7
Magma age (years)	30,000	60,000
Magma temperature (°C)	900	1,100
Heat recovery factor (%)	5	10

Figure 6 shows the resulting graph of MW reserves per km^2 area, plus and minus one standard deviation, as a function of the horizontal distance from the center of the volcano, the total reserves being 425 MW. In a nationwide assessment of geothermal prospects in Nicaragua we applied the proposed methodology to 14 individual volcanoes, ranging in magma volume from 4 to 220 cubic km, magma depth from 3 to 7 km, age from 5,000 to 500,000 years and temperature from 900°C to 1,100°C. The calculated reserves associated with a volcano or volcanic complex ranged from 61 MW to 676 MW.

Concluding Remarks

In the absence of any direct method of estimating the temperature and volume of geothermal reservoirs, the proposed method is a consistent and quantitative methodology for estimating at least the upper limit of geothermal reserves associated with an inadequately explored volcano. Estimating reserves in this way allows one volcanic complex to be compared objectively with another, and makes possible a region-wide inventory of potential geothermal reserves in unexplored or inadequately explored volcanic regions.

The methodology proposed above is perfectly general. For example, instead of equation (1), any other closed-form solution reflecting different magma shapes, and initial and boundary conditions, can be used if warranted. All such closed-form solutions assume a uniform and isotropic medium. If sufficient exploration and drilling have been conducted to define the shape of the magma body and heterogeneity in the medium surrounding the magma, numerical simulation can be conducted instead of using equation (1). In other words, this methodology is equally applicable to thoroughly explored volcanic complexes. As regards the uncertain variables, these can be assigned uniform or triangular or any other type of probability distribution, or can be considered fixed rather than uncertain.

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